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ABSTRACT

The Greek mathematician, Pythagoras, was among the first to undertake a mathematical study of music. His work, resulted in a scale of notes which can produce beautiful melodies and which is easily reproduced in the elementary classroom. In an age when teachers look for an interdisciplinary connection between various aspects of the curriculum, in a day when parents demand much in the way of basic computation, and at a time when students insist on a dazzling presentation to be motivated, Pythagoras' observation and experimentation can provide students with an interesting and all-encompassing utilization of mathematics skills and an inductive approach to learning. Suggested activities include construction and use of a monochord (stretched guitar string), and a "woodwind" prepared from a paper soda straw (as an entertaining drill in fractions). Through measuring, mathematics, and music, these and other activities give students enjoyment, participation, appreciation, and the understanding of the interrelationships of the concepts of mathematics to objects from the real world of their experience. (JN)

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Measurement, Mathematics, and Music

By Katie Blackburn and David White

As primitive man first used his fingers and toes in learning how to count, we can also assume that he used his digits as natural sets of percussion instruments. As simple and as basic a set of materials as fingers, string, and measuring stick can, when used with a few variations upon a theme, be used to introduce music and its mathematical basis to young children.

There are certain simple patterns both in mathematics and in music which demand only a minimum of artistic or factual knowledge for their appreciation as objects of beauty. Fractions and frequency, strings and measuring sticks are based upon a few simple mathematical patterns. Some of these basic patterns were investigated by the noted Greek mathematician, Pythagoras. Besides his contribution of the Pythagorean theorem ($a^2 + b^2 = c^2$), he was among the first to undertake a mathematical study of music. His work resulted in a scale of notes which can produce beautiful melodies and which is easily reproduced in the elementary classroom.

One legend states that Pythagoras, on a long Mediterrean voyage, listened to the music of flapping sails, whistling wind in the rigging, and sighing melodies playing upon the ropes. It was then that he decided to study the connection between the tempest of sound and the vibrating strings. Pythagoras experimented with stretched strings of different length placed under tension. He soon found a definite relationship between the length of the vibrating string and the pitch of the resulting note. He

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discovered that a marvelous and seemingly god-sent connection existed between musical harmony and the whole numbers that we count by every day (1, 2, 3, 4 and so on). Pluck any taut string and a note sounds; then pluck an equally stretched string twice as long and you hear a new note that is one harmonic octave below the first; a string half as long but just as taut will produce a sound that is one octave above the first note. Arrange these strings together and a prototype musical instrument is produced. Hearing such an arrangement for the first time, we can see that it is no wonder the early Greeks gave the status of "god" to Apollo for his invention of the lyre, a simple plucked series of strings of differing lengths.

In an age when teachers look for an interdisciplinary connection between the various aspects of the curriculum, in a day when parents demand much in the way of basic computation, at a time when we know that students insist on a dazzling presentation in order to be motivated, Pythagoras' observation and experimentation can provide students with an interesting and all-encompassing utilization of mathematics skills and an inductive approach to learning. Begin with a simple stretched guitar string. Place the string through two metal eyelets that have been screwed onto a wooden board. When assembled, this simple string arrangement is called a monochord.

Figure 1

Students will note that a musical sound can be made by striking or plucking this string, just as a tuning fork produces a musical note. Striking or plucking causes the adjacent particles of air to vibrate the same rate of vibration as the string or tuning fork. A string that vibrates at 256 times per second produces a pure tone, middle C. Any object that produces this same rate of vibration will produce the tone of middle C. Students can be encouraged to bring in recorded sounds and allowed to present them to their peers who would then guess the rate of vibration (lower or higher than middle C, one octave above or below middle C, and so on). A siren, a metal saw, a humming machine are all good candidates for this initial investigation.

A fine next step for students to take is to build their own monochord. When the monochord has been produced, the students are then ready to investigate the relationship of the length of the string to the sound produced. First, the string of the monochord needs to be stretched by turning the eyelet to obtain the sound of middle C (tuning with an inexpensive tuning fork will do). Now the students are ready to experiment with obtaining various pitches. Using fractional parts of the length of the string (1/2, 3/4, 2/3, and so on), various musical pitches can be obtained. After simply playing around with the concept of relationship of length of string to the sound it produces, students are ready for a more in-depth look.

A pitch pipe, a series of tuning forks, a piano, or recordings of one of these can then be introduced. Using one octave (C, D, E, F, G, A, B, C), students can find the position on the

monochord that corresponds to the sound of these notes. Next, have students measure out the fractional part of the length for each note. Accurate measuring and use of fractions will produce the following relationships of the notes to the length of the string:

C	D	E	F	G	A	B	C
1	8/9	4/5	3/4	2/3	3/5	8/15	1/2

The students are now ready for the vocabulary to discuss their construction. The string moves (vibrates) a certain number of times per second; this is the frequency. The resulting sound is a product of the frequency. The frequency of the note is its relationship to the length of the string. That is, by taking the inverse of the fraction they obtained and multiplying it by the number given to them for the frequency of middle C (256 vibrations per second), they may obtain the frequency of each note they have made on their monochord. Thus, $1 \times 256 = 256$ (middle C), $3/2 \times 256 = 384$ (G), and so on. The string produces a sound whose frequency is therefore the inverse of the length of the entire string; in other words, the frequency (number of vibrations per second) is inversely proportional to the fractions given in the chart above. Middle C vibrates 256 times per second, so to find how many times the other notes vibrate, simply multiply 256 by the inverse of the fraction associated with the note.

Just as the stringed instrument such as the monochord can be used to present music, measuring, and mathematics to children, so too, a simple paper soda straw can easily be used to af-

ford an entertaining drill in fractions. For this activity, each student can be given four or more straws and the following directions:

(1) Cut your first straw so that it will be 8 inches long. Flatten 1/2 inch of one end of the straw and cut off 1/4 inch from each flattened corner. See the diagram.

Figure 2

(2) Cut the second straw so that it will be 4/5 as long, flatten and cut as with the first straw. Cut the third straw 2/3 as long as the first and cut the fourth 1/2 as long as the first. Flatten and cut as before.

(3) To play the straw, put the cut end in your mouth and blow. Put the straws together to form a pitch pipe.

The cut part of the straw acts as the reed in a woodwind. The entire class can be divided into four groups, one group for each straw length. When all play, a major chord can be heard. If just the longest and shortest straw is sounded, an octave can be heard.

Students could also make straws which would, when played, produce the notes as in the monochord. Produce one straw of any given length, then each new straw would be cut in the fractional parts of the monochord: 8/9, 4/5, 3/4, 2/3 and so on.

Figure 3

Additionally, a hardwood stick, when cut into the same fractional relationships as before and dropped on a hard surface such as a desk or a floor, produces pleasing notes that can be arranged and played. For this, either cut or ask the shop teacher to cut pieces of hardwood into lengths measuring 180 mm, 160 mm, 144 mm, 135 mm, 120 mm, 108 mm, and 90 mm respectively. Dowel rods, meter sticks, yardsticks, or even pencils without erasers can be used.

Have the students determine what fractional part of the longest stick the other sticks are. For example, 160 mm is what fractional part of 180 mm? (160/180 = 8/9), 144 mm is what fractional part of 180 mm? (144/180 = 4/5), etc. The students should realize that these fractions are the same as with the monochord. Have the students label the wooden pieces as shown.

Figure 4

Next, ask the students to compare the length of each stick with the next longer one. For example, 90/96 = .94 (rounded to the nearest hundredth), 96/108 = .89, 108/120 = .90, and so on. The students could then prepare the results in a chart and notice the pattern that emerges.

Show the students a picture of a piano keyboard.

Figure 5

If the students compare the ratio of the lengths of the adjacent sticks to the piano keyboard, they can observe that if two white keys have a black key between them, the ratio of the sticks is either .89 or .90. However, if the two white keys do not have a black key separating them, then the ratio of the corresponding sticks is .94. Thus, the whole step ratio is either .89 or .90, and the half step ratio is .94.

In all this work, students can be lead to see the interrelationships between mathematics and various other human activities. Mathematics - the teacher and student will find - is full of surprises, new ideas, and beautiful and pleasing patterns. At the same time, it has a power, a uniqueness, a mystery that may be a fascination for students. In educating students in mathematics, a major goal in the classroom is for the pupil to learn to enjoy and appreciate. We start with simple ideas, relate them to familiar objects, and have students participate in activities. In short, through measuring, mathematics, and music, we give our students enjoyment, participation, appreciation, and the understanding of the interrelationships of the concepts of mathematics to objects from the real world of their experience.

Figure 6

FIGURES

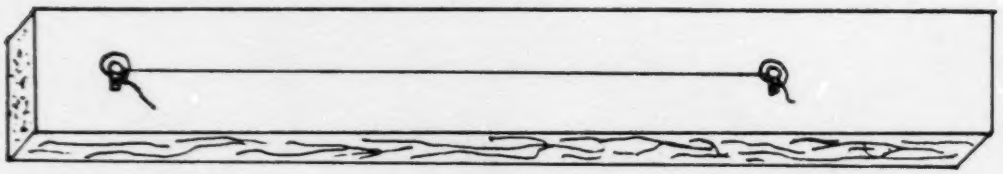

Figure 1

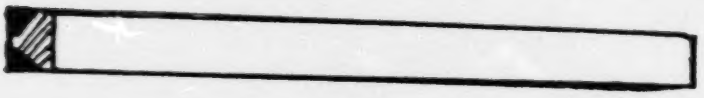

Figure 2

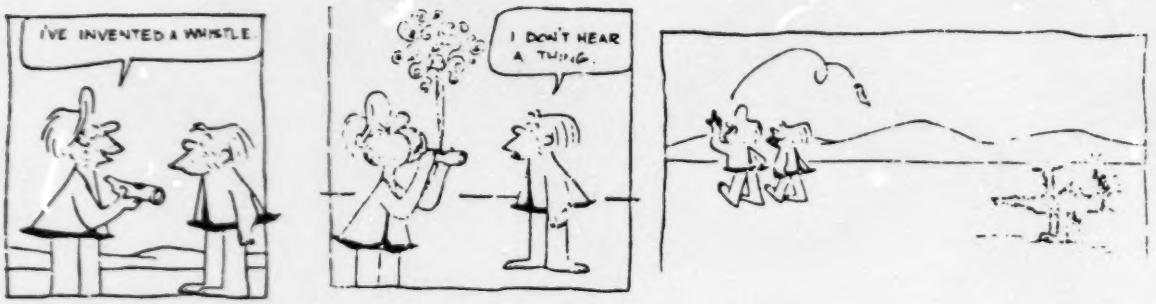

Figure 3

Figure 4

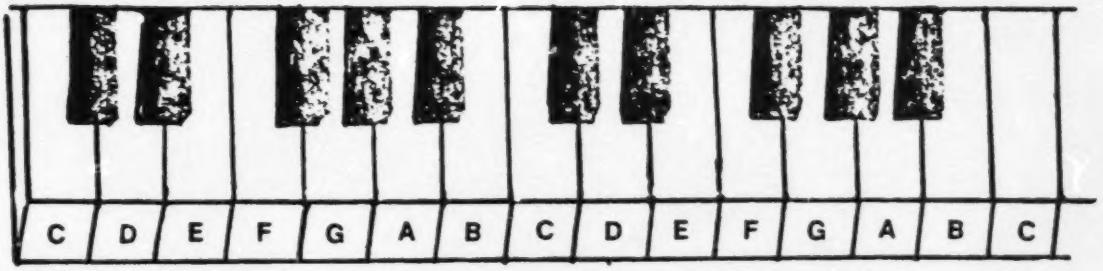

Figure 5

Figure 6